

Neutrino Physics: Fundamentals of Neutrino Oscillations

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In this lecture we review some of the basic properties of neutrinos, in particular their mass and the oscillation behavior. First we discuss how to describe the neutrino mass. Then, under the assumption that neutrinos are massive and mixed, the fundamentals of the neutrino oscillations are discussed with emphasis on subtle aspects which have been overlooked in the past. We then review the terrestrial neutrino oscillation experiments in the framework of three generations of neutrinos with the standard mass hierarchy. Finally, a brief summary of the current status of the solar and atmospheric neutrino problems will be given.

I. INTRODUCTION

Neutrinos are still the least known particles among the well-established fundamental fermions in the Universe. The most important property of the neutrinos is whether or not neutrinos are massive. If they turn out to be massless, one must understand the reason why they are massless. The masslessness of the photon is guaranteed by the local $U(1)_{em}$ gauge invariance. Because of the lack of such a symmetry and in light of the natural occurrence of the massiveness of neutrinos when we extend the standard model of strong and electroweak interactions to accommodate several problems in the model, it is very tempting to assume that neutrinos are indeed massive, although no theory can predict with any certainty the values of neutrino masses. In this lecture, we begin with a brief summary of the current status of many efforts to find the neutrino mass and how to describe the mass [1] [2] [3].

The direct mass measurements based on the decay kinematics of ${}^3\text{H}$, π and τ have steadily been improving but it now appears that we have reached the end of the road on this efforts unless a new generation of the techniques is forthcoming. The current limits are

$$\begin{aligned} m(\nu_e) &\lesssim 4.5 \text{ eV} \\ m(\nu_\mu) &\lesssim 160 \text{ KeV} \\ m(\nu_\tau) &\lesssim 24 \text{ MeV}. \end{aligned} \tag{1}$$

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It is interesting to remark that in the cases of both $m(\nu_e)$ and $m(\nu_\mu)$, all the experimental analyses have consistently yielded *negative* m^2 , which shows that some systematics in experiments are still not properly understood.

On the other hand we have very interesting limits from cosmology. Requiring that neutrinos cannot over-close the Universe, we have

$$\sum_i m_i \lesssim (20 \sim 30) \text{ eV}, \tag{2}$$

where the uncertainty is due to that of the Hubble constant. The above limit applies only when neutrinos are stable or much longer lived than the age of the Universe. The recent COBE data and other astronomical observations provide us with an intriguing possibility that the heaviest neutrino mass might be of order of $1 \sim 10$ eV. A neutrino or neutrinos of this mass range can explain the small scale region of the power spectrum in the form of hot dark matter. Currently, however, the most promising way to probe the neutrino mass is considered to be the neutrino oscillation experiments. The source can be either terrestrial or extraterrestrial.

Indications of massive neutrinos from the oscillation experiments with reactors and accelerators have not been found for many decades with an exception of the recent LSND experiment [4]. We will critically analyze all the reactor and accelerator oscillation experiments using three generations of neutrinos with the standard mass hierarchy. We show that in contrast to the simple two generation analysis, the LSND allowed region lies in the forbidden region of all the previous terrestrial oscillation experiments, when the previous data are analyzed in the framework of three generations of neutrinos with the standard mass hierarchy. Also discussed are implications of the three generation analysis of neutrinoless $\beta\beta$ decay. Finally, we will briefly review the present status of the solar and atmospheric neutrino problems.

II. NEUTRINO MASS

In the past, there have appeared numerous papers on the theory of neutrino mass. For recent references, see, for example, [1], [2] and [3]. None of theories, however, is satisfactory though each theory has its own merit. Basically, there are two ways to generate neutrino masses. First, one modifies the Higgs sector in the standard

model. For example, an additional singlet, doublet or triplet with or without right-handed neutrinos is added to the original Higgs doublet in the standard model. In this case, however, one is forced to introduce a new mass scale in the form of the vacuum expectation value. This, however, is not an explanation of the small neutrino mass. The other possibility is to utilize extremely heavy right-handed neutrinos which appear in models such as left-right symmetry models or GUTS. In the following we list several ways to *describe* the neutrinos mass [1] [5], eventually leading to detailed discussions of the second possibility.

1. Dirac Mass. The simplest way to describe the mass is, of course, to introduce the right-handed neutrinos although they have not been seen so far, at least in the energy region accessible to us now. The mass term in the Lagrangian is

$$\mathcal{L}_{Dirac} = -(\bar{\nu}_L \mathcal{M} \nu_R + \bar{\nu}_R \mathcal{M}^\dagger \nu_L), \quad (3)$$

where $\nu_{L,R}$ is given by

$$\nu_{L,R} = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_{L,R}. \quad (4)$$

In general, \mathcal{M} is a 3×3 complex mass matrix, and hence there is no guarantee that mass eigenvalues are positive. One needs to bi-diagonalize \mathcal{M} using two unitary matrices U and V :

$$U^\dagger \mathcal{M} V = m_D = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \quad (5)$$

where U and V relate the mass eigenstates $\nu_L^{(m)}$ and weak eigenstates ν_L as

$$\begin{aligned} \nu_L &= U \nu_L^{(m)} \\ \nu_R &= V \nu_R^{(m)}. \end{aligned} \quad (6)$$

The diagonalized mass Lagrangian is

$$\mathcal{L}_{Dirac} = -\bar{\nu}_L^{(m)} m_D \nu_R^{(m)} + \text{h.c.} \quad (7)$$

Physically, of course, since only ν_L is involved in weak interactions, the U is the Cabibbo, Kobayashi and Maskawa mixing matrix. In this case, the basic questions to be answered would be where ν_R is and why m_i are so small.

2. Majorana Mass. The Majorana neutrino mass can be described by the use of ν_L alone:

$$\mathcal{L}_{Majorana} = -\frac{1}{2} \bar{\nu}_L^C \mathcal{M} \nu_L + \text{h.c.}, \quad (8)$$

where we note that ν_L^C is a right-handed neutrino. Since

$$\bar{\nu}_L^C \mathcal{M} \nu_L = \bar{\nu}_L^C \mathcal{M}^T \nu_L, \quad (9)$$

we have $\mathcal{M} = \mathcal{M}^T$, i.e. \mathcal{M} is symmetric and diagonalization can be done by a single unitary matrix U in this case. Again, with

$$\nu_L = U \nu_L^{(m)}, \quad (10)$$

Eq.(8) becomes

$$\mathcal{L}_{Majorana} = -\frac{1}{2} [\bar{\nu}_L^C m_D \nu_L + \bar{\nu}_L m_D \nu_L^C]. \quad (11)$$

Defining

$$\nu_{Maj} \equiv \nu_L + \nu_L^C, \quad (12)$$

which is clearly Majorana neutrino, we can rewrite

$$\mathcal{L}_{Majorana} = -\frac{1}{2} \bar{\nu}_{Maj} m_D \nu_{Maj}. \quad (13)$$

Although the expression of Eq.(13) looks similar to that of Eq.(7), there is a fundamental difference between them. In Eq.(13), the lepton number is violated by two units.

3. Dirac–Majorana Mass. First, let us consider the one generation case. The Lagrangian of interest is

$$\mathcal{L}_{D-M} = -M \bar{\nu}_L \nu_R - \frac{1}{2} (m_L \bar{\nu}_L^C \nu_L + m_R \bar{\nu}_L^C \nu_R) + \text{h.c.}, \quad (14)$$

where M is Dirac mass and $m_L(m_R)$ are Majorana masses. If we define a left-handed neutrino state ν as

$$\nu \equiv \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix}, \quad (15)$$

the Dirac–Majorana Lagrangian looks like that of Majorana :

$$\mathcal{L}_{D-M} = -\frac{1}{2} \bar{\nu}^C \mathcal{M} \nu + \text{h.c.}, \quad (16)$$

where the mass matrix \mathcal{M} is

$$\mathcal{M} = \begin{pmatrix} m_L & M \\ M & m_R \end{pmatrix}. \quad (17)$$

It is to be noted that the state ν is not a mass eigenstate. Diagonalizing \mathcal{M} yields

$$m_1 = \frac{1}{2} \sqrt{4M^2 + (m_R - m_L)^2} - \frac{m_L + m_R}{2} \quad (18)$$

$$m_2 = \frac{1}{2} \sqrt{4M^2 + (m_R - m_L)^2} + \frac{m_L + m_R}{2}.$$

Now, the mass eigenstate $\nu^{(m)}$ can be defined as

$$\begin{aligned} \nu^{(m)} &= \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = U \nu \quad (19) \\ &= \begin{pmatrix} \cos \theta \nu_L - \sin \theta \nu_R^C \\ \sin \theta \nu_L + \cos \theta \nu_R^C \end{pmatrix}, \end{aligned}$$

where the mixing angle is given by $\tan 2\theta = 2M/(m_R - m_L)$.

Now let us consider two interesting cases.

- (a) Case with $M \gg m_L, m_R$. In this case m_1 and m_2 are almost degenerate in mass (Eq.(18) implies $m_1 \simeq m_2 \simeq M$) and we have $\theta \simeq 45^\circ$. This case is called special (because $\theta \simeq 45^\circ$) pseudo-Dirac neutrino [6]. In this case, ν_1 and ν_2 have opposite CP phase. And we have half active ν_L and half sterile ν_R^C .
- (b) Case with $m_R \gg M, m_L$. For simplicity, we assume $m_L = 0$. Then, we have

$$\begin{aligned} m_1 &\simeq \frac{M^2}{m_R} \quad (20) \\ m_2 &\simeq m_R, \end{aligned}$$

implying that m_1 is naturally small and m_2 is large. Since $\theta \simeq 0$, ν_L and ν_R^C are practically decoupled. This is the seesaw mechanism [7]. Note that M has the standard model mass scale whereas m_R is the mass scale of the right – handed neutrino associated with models such as L–R symmetry, $SO(10)$, E_6, \dots models.

- 4. Seesaw Mechanism.** In order to apply the seesaw mechanism to practical cases, let us generalize the above to the three generation case by writing

$$m_1 = \frac{m_D^2}{M_R} \rightarrow \bar{m}_1 = \bar{m}_D \frac{1}{M} \bar{m}_D^T, \quad (21)$$

where all barred objects are 3×3 matrices and as before we assume $|(\bar{m}_D)_{ij}| \ll |\bar{M}_{ij}|$. Now the matrix analogous to Eq.(17) is a 6×6 mass matrix \bar{M} given by

$$\bar{M} = \begin{pmatrix} 0 & \bar{m}_D \\ \bar{m}_D & \bar{M}_R \end{pmatrix}. \quad (22)$$

At this point we can consider two possible cases: 1). There is no mass hierarchy among the right-handed neutrinos, i.e.

$$\bar{M}_R \simeq M_R \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (23)$$

implying

$$m(\nu_1) : m(\nu_2) : m(\nu_3) = \frac{m_u^2}{M_R} : \frac{m_c^2}{M_R} : \frac{m_t^2}{M_R}. \quad (24)$$

This is called the quadratic seesaw mechanism.

2). The next possibility is the case where the right-handed neutrinos have the mass hierarchy similar to that of the known "top" quarks, i.e.

$$\bar{M} \simeq M_R \begin{pmatrix} \frac{m_u}{m} & 0 & 0 \\ 0 & \frac{m_c}{m} & 0 \\ 0 & 0 & \frac{m_t}{m} \end{pmatrix}. \quad (25)$$

In this case we have

$$m(\nu_1) : m(\nu_2) : m(\nu_3) = \frac{m_u}{M_R} : \frac{m_c}{M_R} : \frac{m_t}{M_R}. \quad (26)$$

which is called the linear seesaw mechanism.

The above relations as given by Eqs.(24) and (25) are valid at the GUTS scales which means one has to bring them down to the low energy region of our interest using the Renormalization Group Equations (RGE). Considering only the running of the mass in one-loop calculations, for example , Eq.(24) is modified as [8]

$$\begin{aligned} &m(\nu_1) : m(\nu_2) : m(\nu_3) \quad (27) \\ &= \begin{cases} 0.05 \frac{m_u^2}{M_R} : 0.09 \frac{m_c^2}{M_R} : 0.38 \frac{m_t^2}{M_R} & \text{SUSY SU(5)} \\ 0.05 \frac{m_u^2}{M_R} : 0.07 \frac{m_c^2}{M_R} : 0.18 \frac{m_t^2}{M_R} & \text{SO(10)}. \end{cases} \end{aligned}$$

As we can see above, the corrections due to the RGE depend on the models of choice. In the seesaw mechanism, the actual size of neutrino masses will be determined by the mass scale for M_R . In GUTS such as SUSY SU(5), we have

$$\begin{aligned} M_R(\text{mass of } \nu_R) &\sim h V_{GUT} \sim \left(\frac{h}{g} \right) M_X, \quad (28) \\ M_X &\sim g V_{GUT}, \end{aligned}$$

whereas for the SO(10) model, we have

$$M_X \sim \left(\frac{h}{g} \right) M_{L-R}. \quad (29)$$

In the above, h , V_{GUT} , and g are, respectively, the Yukawa coupling, GUTS vacuum expectation value and the gauge coupling constant. It is generally expected that the mass scale of M_R is 10^{10} GeV $\lesssim M_R \lesssim 10^{15}$ GeV , naturally explaining the smallness of the neutrino mass.

III. NEUTRINO OSCILLATIONS

If neutrinos are massive and mixed, neutrinos are produced and detected in the form of the weak eigenstates whereas when they propagate from the point of the production to their detection, their motion is dictated by the mass eigenstates. This leads to the phenomenon of neutrino oscillations [9]. For neutrino oscillations to occur, neutrinos must be massive and mixed. The weak eigenstates and mass eigenstates are related by a unitary matrix U as

$$\nu_W = U \nu_M , \quad (30)$$

where U is parameterized by, in the case of the two generations of neutrinos,

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} . \quad (31)$$

In the above it was assumed that neutrinos are Dirac. For Majorana neutrinos, a CP phase may appear. The equation of motion for the mass eigenstate is

$$i\dot{\nu}_M = H\nu_M = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \nu_M ; \quad \nu_M = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} . \quad (32)$$

Since neutrinos are expected to be extremely relativistic, using the following approximation

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p} : \quad p \simeq E , \quad (33)$$

we have from Eq.(32)

$$i\dot{\nu}_M = \left[E \cdot 1 + \frac{1}{2E} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \right] \nu_M . \quad (34)$$

Replacing $\nu_M \rightarrow e^{i\alpha t} \nu_M$, the above equation becomes

$$i\dot{\nu}_M = (E + \alpha) \nu_M + \frac{1}{2E} \mathcal{M}_M \nu_M , \quad (35)$$

where $\mathcal{M}_M = \text{diag}(m_1^2, m_2^2)$. Taking $\alpha = -E$, we have the equation of motion for ν_M as

$$i\dot{\nu}_M = \frac{1}{2E} \mathcal{M}_M \nu_M . \quad (36)$$

Now, using $\nu_M = U^\dagger(\theta) \nu_W$ (Eq.(30)), the equation of motion for weak eigenstates can be derived as

$$i\dot{\nu}_M = \frac{1}{2E} U(\theta) \mathcal{M}_M U(\theta)^\dagger \nu_W , \quad (37)$$

where

$$\begin{aligned} \mathcal{M}_W &\equiv U(\theta) \mathcal{M}_M U(\theta)^\dagger \quad (38) \\ &= \begin{pmatrix} m_1^2 \cos^2 \theta + m_2^2 \sin^2 \theta & \frac{\Delta}{2} \sin 2\theta \\ \frac{\Delta}{2} \sin 2\theta & m_2^2 \cos^2 \theta + m_1^2 \sin^2 \theta \end{pmatrix} . \end{aligned}$$

In Eq.(38),

$$\begin{aligned} \Delta &\equiv m_2^2 - m_1^2 \\ \langle m(\nu_e)^2 \rangle &\equiv m_1^2 \cos^2 \theta + m_2^2 \sin^2 \theta \\ \langle m(\nu_\mu)^2 \rangle &\equiv m_2^2 \cos^2 \theta + m_1^2 \sin^2 \theta . \end{aligned} \quad (39)$$

By solving the above differential equation, one can easily obtain the well-known oscillation probability for the two generation case. For example,

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2(2\theta) \sin^2(\Delta L / 4E_\nu) . \quad (40)$$

A similar treatment for the three generation case can be trivially performed but it is not very informative. Since we shall discuss three generation cases below, we close this section with the case of two generations.

IV. WAVE PACKET EFFECT

The above treatment of the neutrino oscillations assumes explicitly that the neutrinos that are produced via weak interactions are described by plane waves. However, in practice since neutrino-emitting particles are never in a free stable state, neutrinos cannot be described by plane waves. Rather, they must be described by wave packets, due to pressure broadening or collisions of neutrino-emitting particles [10]. Therefore, the size of wave packet σ_x is given by dimension of the region within which production processes are effectively localized

$$\sigma_x \sim \frac{\text{mean free path } (l)}{\text{mean thermal velocity } (v)} \sim \frac{T^{\frac{3}{2}}}{N} , \quad (41)$$

where we have used

$$\begin{aligned} l &\sim \frac{T^2}{N} \\ v &\sim T^{\frac{1}{2}} . \end{aligned} \quad (42)$$

It is easy to see that $\sigma_x \rightarrow \infty$ as $l \rightarrow \infty$ and $v \rightarrow 0$, recovering the case of plane waves. Let us consider several examples in order to see how large or small the size of σ_x is.

- Solar Core.** For neutrinos emitted from the process ${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$, we have typically $T \sim 1.3$ KeV and $\rho \sim 120g/cm^3$, implying $\sigma_x \sim 10^{-7}cm$. For neutrinos emitted from the process $p + p \rightarrow {}^2H + e^+ + \nu_e$, one has typically $T \sim 1.1$ KeV and $\rho \sim 100g/cm^3$, leading to $\sigma_x \sim 5 \times 10^{-7}cm$.

- Nuclear Reactor.** In the case of neutrinos from nuclear reactors, we have $v \sim P_N/M_N \sim 10^{-5}$, $N \sim 10^{23}/cm^3$ and $l_N \sim 10^{-7}cm$, leading to $\sigma_x \sim 10^{-4}cm$.

3. Accelerator/Atmospheric Neutrinos. Since they are decay-products of π , or μ , σ_x is characterized by the typical weak interaction length, t_W , so that $\sigma_x \sim ct_W \gtrsim 10^2 \text{cm}$.

4. Supernova Neutrinos. Neutrinos from the core with $T \sim 10 \text{ MeV}$ and $\rho \sim 10^{14} \text{g/cm}^3$ have $\sigma_x \sim 10^{-13} \text{cm}$. Neutrinos from the neutrino sphere with $T \sim 1 \text{ MeV}$ and $\rho \sim 100 \text{g/cm}^3$ have $\sigma_x \sim 10^{-9} \text{cm}$.

What do wave packets do to neutrino oscillations? Recalling that a plane wave is described by $|\psi_a(x, t) \rangle = e^{ipx - E_a t} |\nu_a \rangle$ with $E_a = \sqrt{p^2 + m_a^2} \simeq p + m_a^2/2p$, and using the wave packet in the momentum space

$$\psi_a(p) = \left(\frac{1}{\sqrt{2\pi}\sigma_p} \right)^{\frac{1}{2}} e^{-\frac{(p-p_a)^2}{4\sigma_p^2}}, \quad (43)$$

the wave packet in the space is described by, with $\sigma_p \sigma_x \sim 1$,

$$|\psi_a(x, t) \rangle = \left(\frac{1}{\sqrt{2\pi}\sigma_p} \right)^{\frac{1}{2}} e^{ipx - E_a t - \frac{(x-v_a t)}{4\sigma_x^2}} |\nu_a \rangle. \quad (44)$$

By using this wave packet in a way similar to the standard treatment of neutrino oscillations, instead of the usual expression for the plane wave,

$$P(\nu_e \rightarrow \nu_\mu) \sim \sum_{a,b} U_{\nu_{\mu a}} U_{\nu e b}^* U_{\nu_{\mu b}}^* U_{\nu e b} e^{i \frac{2\pi L}{L_{a,b}^{osc}}}, \quad (45)$$

one finds, for wave packets [10],

$$P(\nu_e \rightarrow \nu_\mu) \sim \sum_{a,b} U_{\nu_{\mu a}} U_{\nu e b}^* U_{\nu_{\mu b}}^* U_{\nu e b} e^{i \frac{2\pi L}{L_{a,b}^{osc}} e^{-\frac{L}{L_{a,b}^{coh}}}}, \quad (46)$$

where

$$L_{a,b}^{osc} \equiv \frac{4\pi E}{m_a^2 - m_b^2} \quad (47)$$

is the oscillation length and

$$L_{a,b}^{coh} \equiv 4\sqrt{2}E^2 \frac{\sigma_x}{m_a^2 - m_b^2} \quad (48)$$

is the coherence length beyond which neutrinos do not oscillate in contrast to the case of plane waves. Therefore, the wave packets effects introduce, as can be seen above, an exponentially damping factor which is present only when $a \neq b$, implying that the oscillating term would eventually disappear beyond the coherence length. Typical coherence lengths based on the above mentioned sizes of σ_x are:

1. **Solar Neutrinos:** $L^{coh} \sim 10^8 \text{cm}$.
2. **Reactor Neutrinos:** $L^{coh} \sim 10^9 \text{cm}$.
3. **Accelerator Neutrinos:** $L^{coh} \sim 10^{21} \text{cm}$.

4. Supernova Neutrinos: $L^{coh} \sim 100 \text{cm}$.

The above numbers are based on typical values of E_ν and $\Delta m^2 \sim 1 \text{eV}^2$ in their respective cases. Therefore, in the usual reactor and accelerator experiments, unless Δm^2 turn out to be unexpectedly large the oscillation lengths are long enough so that one can indeed observe oscillations. Note, however, that in the case of supernova neutrinos, the coherence length is so short that neutrinos from supernova do not oscillate on the way to the Earth. This, however, does not mean that there occur no flavor changes among three generations of neutrinos. They simply do not have oscillating terms in the transition probability.

V. WEAK AND MASS EIGENSTATES

The question we like to ask in this Section is whether or not the weak eigenstates make sense. In the standard treatment of neutrino oscillations, one always uses the weak eigenstates to discuss oscillations in vacuum and in matter. But, here we will show that strictly speaking, the weak eigenstates do not make sense, although the weak eigen fields are well-defined [11]. In order to illustrate the point, let us consider the detection of ν_e via $\nu_e + n \rightarrow p + e^-$. The amplitude of interest is

$$\begin{aligned} \frac{G_F}{\sqrt{2}} \langle e^-, p | \bar{e} \gamma^\mu (1 - \gamma_5) \nu_e J_\mu^{(h)}(m_i) | \nu_\alpha, n \rangle \\ = \sum_i U_{ei} U_{\alpha i}^* \left[\langle e^- | \bar{e} \gamma^\mu (1 - \gamma_5) \nu_i | \nu_i \rangle h_\mu^{(i)}(n, p) \right], \end{aligned} \quad (49)$$

where the equality was obtained by using

$$\begin{aligned} \nu_e &= \sum_i U_{ei} \nu_i \\ |\nu_\alpha\rangle &= \sum_i U_{\alpha i}^* |\nu_i\rangle. \end{aligned} \quad (50)$$

The crucial observation here is that we deliberately set ν_α instead of ν_e . We do know from the definition of the amplitude on the left-hand side that it is zero unless $\alpha = e$. However, the right hand side of Eq.(48) is not necessarily zero even if $\alpha = e$ is satisfied, since unless the quantity in the square bracket is factored out from the sum, we do not have the orthonormality

$$\sum_i U_{ei} U_{\alpha i}^* = \delta_{\alpha e}. \quad (51)$$

The factorization is possible only when neutrinos are relativistic and the hadron part is not affected by the presence of the neutrino mass. This simple observation clearly indicates that unless neutrinos turn out to be always relativistic, a caution is needed when one uses the concept of the weak eigenstates. In fact, it has been shown that the Fock space of the weak eigenstates $|\nu_\alpha\rangle$

does not exist [11]. The Fock space can be shown to exist only in the limit of the relativistic neutrinos. Hence, only if and when neutrinos are non-relativistic, one is allowed to calculate the standard oscillation probability without using the concept of the weak eigenstates. A rigorous calculation without resort to the weak eigenstates has already carried out [12]. One can demonstrate that a specific production and detection process of certain weak eigenstate neutrinos can be expressed simply by using the mass eigenstates, and only in the extreme relativistic case, the well-known transition probability can be factored out from the expression.

VI. TERRESTRIAL NEUTRINO OSCILLATIONS

In this Section, we will review the latest data on the reactor and accelerator oscillation experiments in the framework in which the three masses, m_1 , m_2 and m_3 of the mass eigenstates ν_1 , ν_2 and ν_3 are ordered in such a way as suggested by the most popular mechanism that can explain the smallness of the neutrino mass, i.e. see-saw mechanism [7],

$$m_1 \ll m_2 \ll m_3. \quad (52)$$

An additional assumption is that the the solar neutrino problem is real and the deficit is at least 10 percent or more of the standard solar model prediction. The details on the deficits of the four current experiments are not important for our argument. In any case we need to assume

$$\Delta m_{21}^2 \lesssim 10^{-3} \text{ eV}^2. \quad (53)$$

Of course we are also assuming the standard mixing denoted by the unitary transformation

$$\nu_\alpha = \sum_{i=1}^3 U_{\alpha i} \nu_i, \quad (54)$$

where $\alpha = e, \nu$, and τ and ν_i is a mass eigenfield with mass m_i . (The arguments presented here are based on the two papers [13] and [14].) The environment of the terrestrial oscillation experiments is summarized by the following ranges of the distances (L) between the source and the detector and the neutrino energies(E_ν)

$$\begin{aligned} L &\simeq 10\text{m} \sim 1\text{Km}, \\ E_\nu &\simeq 1\text{MeV} \sim 10\text{GeV}. \end{aligned} \quad (55)$$

Because of the assumption of the mass hierarchy as assumed in Eq.(52), we have two independent values of Δm^2 , i.e. Δm_{21}^2 and Δm_{31}^2 . For the terrestrial neutrino oscillations, we can see with Eq.(40) that

$$\sin^2\left(\frac{\Delta m_{21}^2 L}{4E_\nu}\right) \simeq 0, \quad (56)$$

which implies that there remains only one mass scale in the scheme, i.e. $\Delta m_{31}^2 \equiv \Delta m^2$.

Under these assumptions, the transition probability for the oscillation $\nu_\alpha \rightarrow \nu_\beta$ is given by

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= \\ &\left| U_{\beta 1} U_{\alpha 1} + U_{\beta 2} U_{\alpha 2} + U_{\beta 3} U_{\alpha 3} \exp\left\{-\frac{i \Delta m_{31}^2 L}{2E_\nu}\right\} \right|^2. \end{aligned} \quad (57)$$

Note that there appears only one oscillation term in Eq.(57). From Eq.(57), we have ,for example,

$$P(\nu_\mu \rightarrow \nu_e) = A(\nu_\mu \rightarrow \nu_e) \sin^2\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right), \quad (58)$$

$$P(\nu_\mu \rightarrow \nu_\tau) = A(\nu_\mu \rightarrow \nu_\tau) \sin^2\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right),$$

where the oscillation factors A's are given by

$$A(\nu_\mu \rightarrow \nu_e) = 4|U_{\mu 3}|^2 |U_{e 3}|^2, \quad (59)$$

$$A(\nu_\mu \rightarrow \nu_\tau) = 4|U_{\mu 3}|^2 |U_{\tau 3}|^2 = 4|U_{\mu 3}|^2 (1 - |U_{e 3}|^2 - |U_{\mu 3}|^2).$$

It is to be emphasized here that in their appearance, the oscillation probabilities given in Eq.(58) take the same form as the two generation case. In fact, the two generation cases are reproduced when $A(\nu_\mu \rightarrow \nu_e)$ and $A(\nu_\mu \rightarrow \nu_\tau)$ are replaced by $\sin^2(2\theta_{e\mu})$ and $\sin^2(2\theta_{\mu\tau})$, respectively. However, physical implications are quite different. Namely, in the case of two generations the two probabilities have nothing to do with each other and are not related in any way. But, in the three generation case, two probabilities are inter-related since both are determined by the same $|U_{\mu 3}|^2$ and $|U_{e 3}|^2$.

Similarly, the survival probabilities $P(\nu_\alpha \rightarrow \nu_\alpha)$ are given by

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - B(\nu_\alpha \rightarrow \nu_\alpha) \sin^2\left(\frac{\Delta m^2}{4E_\nu}\right), \quad (60)$$

where the oscillation factor $B(\nu_\alpha \rightarrow \nu_\alpha)$ is given by

$$B(\nu_\alpha \rightarrow \nu_\alpha) = 4|U_{\alpha 3}|^2 (1 - |U_{\alpha 3}|^2)^2. \quad (61)$$

Therefore, all the oscillation probabilities are completely determined by the three parameters

$$\Delta m_{31}^2 \equiv \Delta m^2, \quad |U_{e 3}|^2, \quad \text{and} \quad |U_{\mu 3}|^2 \quad (62)$$

and oscillation probabilities are all inter-dependent. This is simply not the case in the two generation case. Furthermore, in our model, we have

$$P(\nu_\alpha \rightarrow \nu_\alpha) = P(\nu_{\bar{\alpha}} \rightarrow \nu_{\bar{\alpha}}), \quad (63)$$

without the assumption of CP conservation.

We now proceed to analyze the data using the formulas derived above. The data to be used in our analysis are as follows: Bugey [18] $\nu_e \rightarrow \nu_x$, CDHS, [19] and CCFR84

[20] $\nu_\mu \rightarrow \nu_x$ and BNL E776 [21], KARMEN [22] and LSND [4] $\nu_\mu \rightarrow \nu_e$ experiments.

First, from the disappearance (or survival) experiments, we can find limits on the oscillation amplitudes B 's as

$$\begin{aligned} B(\nu_e \rightarrow \nu_e) &\lesssim B^0(\nu_e), \\ B(\nu_\mu \rightarrow \nu_\mu) &\lesssim B^0(\nu_\mu), \end{aligned} \quad (64)$$

where B^0 's are obtained from the data for fixed values of Δm^2 and are, in general, small numbers since no clear disappearance of the initial neutrinos has been observed as yet. Using Eq.(16), and solving it for $|U_{\alpha 3}|^2$, we find the following two bounds

$$\begin{aligned} U_{\alpha 3} &\lesssim \frac{(1 - \sqrt{1 + B^0(\nu_\alpha)})}{2} : \text{small}, \\ U_{\alpha 3} &\gtrsim \frac{(1 + \sqrt{1 - B^0(\nu_\alpha)})}{2} : \text{close to one}. \end{aligned} \quad (65)$$

Therefore, there are four possibilities.

1. Region I. Small $|U_{e3}|^2$ and $|U_{\mu 3}|^2$,
2. RegionII. Small $|U_{e3}|^2$ and large $|U_{\mu 3}|^2$,
3. Region III. Large $|U_{e3}|^2$ and small $|U_{\mu 3}|$,
4. Region IV. Large $|U_{e3}|^2$ and $|U_{\mu 3}|^2$,

where *large* means close to one but less than or equal to one. First, it is obvious that the region IV is not allowed simply because of the unitarity condition. The region III is also excluded because with $|U_{e3}|^2$ obtained from the oscillation data, the solar neutrinos are depleted by only 10 percents, which is in sharp contrast to what has been observed. Thus, we are left with two regions only. To be more specific, in the model under consideration, the survival probability of the solar neutrinos (ν_e) is given by [13]

$$P(\nu_e \rightarrow \nu_e) = (1 - |U_{e3}|^2)^2 P^{(1,2)}(\nu_e \rightarrow \nu_e) + |U_{e3}|^4, \quad (66)$$

where $P^{(1,2)}(\nu_e \rightarrow \nu_e)$ is the survival probability due to the mixing between the first and the second generations. If the parameter $|U_{e3}|^2$ is large, we have, for all values of the neutrino energy, $P(\nu_e \rightarrow \nu_e) \gtrsim 0.92$, which is not compatible with the results of the current solar neutrino experiments. It is worth commenting that the case I is consistent with the standard mass hierarchy

$$\langle m(\nu_e) \rangle \ll \langle m(\nu_\mu) \rangle \ll \langle m(\nu_\tau) \rangle, \quad (67)$$

whereas the case II corresponds to an inverted hierarchy

$$\langle m(\nu_e) \rangle \ll \langle m(\nu_\tau) \rangle \ll \langle m(\nu_\mu) \rangle. \quad (68)$$

Although it is quite unnatural, this possibility is not excluded in this analysis.

A. Region I

The region I is consistent with the standard mass hierarchy of the *effective* masses of the weak eigenstates. We will consider the limits on the oscillation amplitude $A(\nu_\mu \rightarrow \nu_e)$. If we simply take the negative results of BNL E776 [21] and KARMEN [22] presented in the two generation form, the limits on the oscillation amplitude $A(\nu_\mu \rightarrow \nu_e)$ would be the same as those obtained in the two generation analysis. If the data are interpreted in this way, some part of the allowed region in the $\Delta m^2 - \sin^2(2\theta)$ plot obtained by the positive result of the LSND experiment is not ruled out by the previous negative results. However, as emphasized earlier, we can find additional constraints on $A(\nu_\mu \rightarrow \nu_e)$ since this oscillation channel is related to other oscillations such as $\nu_\mu \rightarrow \nu_\tau$ and $\nu_e \rightarrow \nu_\tau$. For example, we have

$$\begin{aligned} A(\nu_e \rightarrow \nu_\tau) &= 4 |U_{e3}|^2 |U_{\tau 3}|^2 \simeq 4 |U_{e3}|, \\ A(\nu_\mu \rightarrow \nu_\tau) &= 4 |U_{\mu 3}|^2 |U_{\tau 3}|^2 \simeq 4 |U_{\mu 3}|. \end{aligned} \quad (69)$$

In addition, we have

$$A(\nu_\mu \rightarrow \nu_e) = 4 |U_{e3}|^2 |U_{\mu 3}|^2 \simeq \frac{A(\nu_e \rightarrow \nu_\tau) A(\nu_\mu \rightarrow \nu_\tau)}{4}. \quad (70)$$

The above relations provide us with much more stringent constraints on the $A(\nu_\mu \rightarrow \nu_e)$ than the ones from direct two generation analyses or disappearance data. Using the negative results of the $\nu_e \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_\tau$ oscillation experiments, we can obtain the forbidden region in the $\Delta m^2 - A(\nu_\mu \rightarrow \nu_e)$ plot which turns out to be much wider than the simple two generation result and the allowed region by the LSND experiment happens to be well within this forbidden region. That is, the LSND result is not consistent with the previous negative oscillation experiments if the mass hierarchy is the standard one.

B. Region II

This region can be a solution because of the quadratic nature of the relation(16) in $|U_{\alpha i}|^2$. In this region, $|U_{e3}|^2$ and $|U_{\tau 3}|^2$ are small and $|U_{\mu 3}|^2$ is close to one (but less than or equal to one). This implies that the standard mass hierarchy of effective weak eigenstate masses is inverted, i.e.,

$$\langle m(\nu_e) \rangle \ll \langle m(\nu_\tau) \rangle \ll \langle m(\nu_\mu) \rangle. \quad (71)$$

First, as in the case of the Region I, we have direct limits on $A(\nu_\mu \rightarrow \nu_e)$ from the negative results of the $\nu_\mu \rightarrow \nu_e$ experiment of BNL E776 [21], which yields limits on $|U_{e3}|^2$ since in this case $|U_{\mu 3}|^2$ is close to one. On the other hand, the positive indication of the LSND experiment sets the bounds

$$A^-(\nu_\mu \rightarrow \nu_e) \leq A(\nu_\mu \rightarrow \nu_e) \leq A^+(\nu_\mu \rightarrow \nu_e), \quad (72)$$

where $A^-(\nu_\mu \rightarrow \nu_e)$ and $A^+(\nu_\mu \rightarrow \nu_e)$ are, respectively, the bounds for fixed values of Δm^2 . The bounds on $|U_{e3}|^2$ can be found from Eq.(26). Also, in this case we can find the following relation

$$A(\nu_\tau \rightarrow \nu_e) = A(\nu_\mu \rightarrow \nu_e)A(\nu_\mu \rightarrow \nu_\tau)/4 \quad (73)$$

which shows that the oscillation $\nu_e \rightarrow \nu_\tau$ is significantly suppressed. In contrast to the previous case of the standard mass hierarchy, the $A(\nu_\mu \rightarrow \nu_e)$ is not constrained any further by other oscillation experiments. Therefore, the LSND result is not in contradiction with the previous experiments.

C. Neutrinoless $\beta\beta$ Decay

We have shown above that in the framework of the model with the mass hierarchy, the data on the terrestrial oscillation experiments and the solar neutrinos indicate that the mixing parameter $|U_{e3}|^2$ is small. If massive neutrinos are indeed Majorana particles, this result has important implications on $(\beta\beta)_{0\nu}$ decay experiments. The Dirac and Majorana nature of neutrinos can best be tested or answered in the experiment of neutrinoless double beta decays. Even after heroic efforts by countless experimentalists, no such decay has so far been observed. This process is possible only if neutrinos are Majorana. The Majorana neutrinos inherently violate the lepton number by two units which is a necessary condition for neutrinoless double beta-decay process to occur. Its rate is proportional to, in the absence of the right-handed coupling, the square of an effective mass [1] [5] [15] [16]

$$\langle m_\nu \rangle = \left| \sum_i U_{ei}^2 m_i \eta_i \right| \quad (74)$$

where $\eta_i = \pm 1$ is the CP phase of the Majorana neutrino ν_i . So far no $(\beta\beta)_{0\nu}$ decay has been observed, setting limits on $\langle m_\nu \rangle \lesssim 1\text{eV}$ [15] [16] [17].

Because of the assumed mass hierarchy of the mass eigenstates, we have, from Eq.(28),

$$\langle m_\nu \rangle \simeq |U_{e3}|^2 m_3 \simeq |U_{e3}|^2 \sqrt{\Delta m^2}. \quad (75)$$

In the Region I, both $|U_{e3}|^2$ and $|U_{\mu 3}|^2$ are small and $|U_{e3}|^2$ is constrained simply by the Bugey [18] experiment alone for the interval of the experiment $10^{-1}\text{eV}^2 \lesssim \Delta m^2 \lesssim 10^3 \text{ eV}^2$. For example, for $\Delta m^2 \leq 5 \text{ eV}^2$, we have

$$\langle m_\nu \rangle \lesssim 10^{-1}\text{eV}, \quad (76)$$

which implies that we need the sensitivity of the next-generation experiment on $(\beta\beta)_{0\nu}$ decay.

On the other hand, $|U_{e3}|^2$ is severely constrained, in the region II, by BNL E776 and LSND experiments. For a wide range of Δm^2 , we have

$$\langle m_\nu \rangle \lesssim 10^{-2}\text{eV}, \quad (77)$$

implying that if the Region II turns out to be the right region, the observation of $(\beta\beta)_{0\nu}$ decay becomes a formidable, if not impossible, task. We emphasize that in this case the LSND results are not completely ruled out by the previous oscillation experiments.

We can entertain some other possibilities in the mass hierarchy besides the standard mass hierarchy assumed here [14]. For example, we can assume

$$m_1 \ll m_2 \simeq m_3. \quad (78)$$

This scheme was recently considered in [23]. In favor of such a scheme, there are some cosmological arguments and astrophysical arguments concerning the r -process production of heavy elements in the neutrino-heated ejecta of supernova. In this case, it can be shown [14] that

$$\langle m \rangle \simeq m_3. \quad (79)$$

In this case, neutrinoless double beta decay experiments can directly probe the mass of ν_3 .

VII. SOLAR AND ATMOSPHERIC NEUTRINOS

Perhaps, the most intriguing indication, at present, for the massive neutrino comes from the solar neutrinos and to lesser extent from the atmospheric neutrinos. The all four solar neutrino experiments, Kamiokande [24], GALLEX [25], SAGE [26] and Homestake [27], have seen the deficit of the expected rates based on the so-called standard solar model [28]. We list the latest experimental results together with the predictions of the standard solar model of Bahcall and Pinsonneault [28].

1. Kamiokande:

$$\Phi_{expt} = \frac{(2.89 \pm 0.2 \pm 0.35) \times 10^6}{\text{cm}^2\text{sec}}, \quad (80)$$

$$\Phi_{theor} = \frac{(6.62 \pm 0.7) \times 10^6}{\text{cm}^2\text{sec}}.$$

2. GALLEX, SAGE:

$$\Sigma_{expt} = 74 \pm 8 \text{ SNU}, \quad (81)$$

$$\Sigma_{theor} = 132 \pm 7 \text{ SNU}.$$

3. Homestake:

$$\Sigma_{expt} = 2.55 \pm 0.25 \text{ SNU}, \quad (82)$$

$$\Sigma_{theor} = 8.1 \pm 1.0 \text{ SNU}.$$

We note that the amount of the deficit is different for different experiments. It has also been demonstrated that the deficit is most prominent for the medium energy neutrinos which come mainly from the ^{7}Be and CNO reactions. This feature is almost impossible to explain away with the changes in relevant parameters for the Sun. Therefore, in addition to somewhat different amounts of the deficit in the four experiments, one must explain the disappearance of the medium energy neutrinos from the Sun. This can be nicely done with the MSW effects [29].

The MSW effects are the results of a peculiar resonance behavior of neutrinos in matter. When neutrinos pass through a medium, they see a potential V due to their coherent interactions with the particles in matter. Since this potential is linearly proportional to G_F , the effects can be large if matter density is sufficiently large. Neglecting the contribution from neutral current interactions, which does not play any role in the discussion of the MSW effect, the effective potential V induces an increase (in the case of ν_e in the ordinary matter) of $2E_\nu V$ in the mass squared of m_{ν_e} , which is $m_1^2 \cos^2 \theta + m_2^2 \sin^2 \theta$ in vacuum. This is due to the fact that weak eigenstate neutrinos see the effective potential, not the mass eigenstates. As a consequence, the equation of motion of neutrinos in matter becomes Eq.(37) with Eq.(38) with the one-one element replaced by a new matter value which is $m_1^2 \cos^2 \theta + m_2^2 \sin^2 \theta + 2E_\nu V$. Diagonalizing the matrix in Eq.(38) with the above modification, one can find how neutrino masses and mixing angles are effectively modified in matter. Changes in them are such that when certain resonance conditions among Δm^2 and the vacuum mixing angle are met and at the same time changes in matter density are smooth, ν_e is very efficiently converted into, in the case of two generations of neutrinos, ν_μ . This is called the adiabatic process. If the change in density is not smooth, the conversion is not very efficient and called the non-adiabatic process. Whether the process is adiabatic or not depends very crucially on the neutrino energy. This is where the energy dependence of the ν_e conversion probability comes in. We note here that the above four experiments have different energy thresholds (GALLEX and SAGE have the same threshold). If the solar neutrinos are depleted according to the way the MSW effects are in operation, one can find two possible sets of solutions for Δm^2 and $\sin^2(2\theta)$. One is the small angle solution, the other being the large angle solution. It is often said in the literature that the fit with small angle is better but this conclusion is based on the two generation analysis and the large angle solution is as good as the small angle solution. In any case, we shall discuss the small angle solution only in this article. The 90 percent C.L. solution of the all four experiments is given by

$$\begin{aligned} \Delta m^2 &= m_2^2 - m_1^2 \simeq 10^{-5} \\ \sin^2(2\theta) &\simeq 5 \times 10^{-3}. \end{aligned} \quad (83)$$

Assuming the standard mass hierarchy of $m_2 \gg m_1$, the above gives

$$m_2 \simeq 3 \times 10^{-3} \text{ eV}, \quad (84)$$

Identifying this mass with $0.09m_c^2/M_R$ as given by the quadratic seesaw mechanism (Eq.(27)) with SUSY SU(5), we find $M_R \simeq 10^{11}$ GeV. Given M_R , one finds $m_1 \simeq 10^{-8}$ eV and $m_3 \simeq 10$ eV.

The vacuum oscillation explanation of the solar neutrino puzzle is also possible but it is marginally successful at present. In particular, the absence of the medium energy solar neutrinos is non-trivial to explain with the vacuum oscillation although it is not impossible. In general, the vacuum oscillation solution yields smaller neutrino masses than those determined by the MSW effect solution. For example, it gives $m_2 \simeq 10^{-5}$ eV. Therefore, the seesaw mechanism predictions for m_1 and m_2 are accordingly smaller.

The atmospheric neutrino problem has recently been getting serious. The atmospheric neutrinos used to be unwanted backgrounds for the proton decay experiments but they themselves have become the subject of important study. With an exception of the Frejus experiment, Kamiokande, IMB, Soudan II and MACRO all see the muon deficit. In order to reduce uncertainties coming from those of calculations, it is customary to consider the ratios of ratios, $R \equiv (\frac{N_\mu}{N_e})_{\text{expt}} / (\frac{N_\mu}{N_e})_{\text{theor}}$, where $N_{\mu(e)}$ is the number of muon (electron) events induced by $\nu_\mu(\nu_e)$. The following is the summary of the recent experimental results.

$$\begin{aligned} R(\text{Kamioka}) &= 0.60 \pm 0.06 \pm 0.05 \\ R(\text{IMB}) &= 0.56 \pm 0.04 \pm 0.04 \\ R(\text{Soudan II}) &= 0.75 \pm 0.16 \pm 0.14 \\ R(\text{MACRO}) &= 0.73 \pm 0.06 \pm 0.12 \\ R(\text{Frejus}) &= 0.99 \pm 0.13 \pm 0.08. \end{aligned} \quad (85)$$

Currently, the most popular interpretation of the anomaly is that assuming the experiments (with an exception of Frejus) are right, ν_μ 's in the atmospheric neutrinos with the expected excess of a factor of two compared to ν_e , are somehow being depleted on the way to the detectors. This depletion can be attributed to the oscillation of ν_μ into ν_τ . This conclusion is based on the observation that while νe 's do not have enough length (due to small Δm^2) to travel to oscillate into ν_μ or ν_τ , the distances for ν_μ to travel are just enough (due to large Δm^2) so that ν_μ can oscillate partially into ν_τ . If this interpretation turns out to be correct, the following set of the parameters can explain the observed deficits

$$\begin{aligned} \Delta m_{3,2}^2 &\equiv m_3^2 - m_2^2 \simeq 10^{-2} \text{ eV}^2 \\ \sin^2(2\theta) &\simeq 1. \end{aligned} \quad (86)$$

Again in the spirit of the mass hierarchy, it means that $m_3 \simeq 10^{-1}$ eV. This is in contradiction with the LSND

result(because the LSND gives $\Delta m_{2,1}^2 \simeq 1\text{eV}$, which implies $m_2 \simeq \text{eV}$) and it also means that neutrinos can not play a significant role as hot dark matter. One must obviously wait for better data for the atmospheric neutrinos in order to draw a firm conclusion one way or the other. We conclude by saying that the LSND issue and the atmospheric neutrino problem can conclusively be settled by planned long-baseline experiments such as Fermilab to Soudan with $L = 730$ Km, CERN to Gran Sasso with $L = 720$ Km and KEK to SuperKamiokande with $L = 250$ Km. The region to be explored by these long-baseline experiments in the $\Delta m^2 - \sin^2(2\theta)$ plot is precisely the region relevant to the LSND and the atmospheric neutrino issues.

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- [1] C.W. Kim and A. Pevsner, "Neutrinos in Physics and Astrophysics", Contemporary Concepts in Physics, Vol.8 (Harwood Academic, Gordon and Breach, Chur, Switzerland,1993).
 - [2] F. Bohm and P. Vogel, "Physics of Massive Neutronos" (Cambridge University Press, Cambridge,1987).
 - [3] R.N. Mohapatra and P.B. Pal,"Massive Neutrinos in Physics and Astrophysics" (World Scientific, Singapore,1991).
 - [4] C. Athanassopoulos et al., Phys. Rev. Lett. **D75**, 2650 (1995).
 - [5] S. M. Bilenky and S. T. Petcov, Rev. of Mod. Phys. **59**, 671 (1987).
 - [6] L. Wolfenstein, Nucl. Phys. **B186**, 147 (1981).
 - [7] M. Gell-Mann, P. Ramond and R. Slansky,in "Supergravity", ed F. van Nieuwenhuizen and D. Freedman, (North Holland, Amsterdam,1979) p. 315; T. Yanagida, Proc. of the Workshop on Unified Theory and the Baryon Number of the Universe, KEK,Japan, 1979.
 - [8] S. Bludman, D. Kennedy and P. Langacker, Phys. Rev.**D45**, 1810 (1992).
 - [9] B. Pontecorvo, JETP, **6**, 429 (1958); Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).
 - [10] C. Giunti, C. W. Kim and U. W. Lee, Phys. Rev. **D44**, 3635 (1991).

- [11] C. Giunti, C. W. Kim and U. W. Lee, Phys. Rev. **D45**, 1231 (1992).
- [12] C. Giunti, C. W. Kim, J.A. Lee and U. W. Lee, Phys. Rev. **D48**, 4310 (1993).
- [13] "Short Base-Line Neutrino Oscillations and Neutrinoless Double Beta Decay in the Framework of Three Neutrino Mixing and a Mass Heirarchy", S.M. Bilenky, A. Bottino, C. Giunti and C. W. Kim, to be published.
- [14] "Short Base-Line Neutrino Oscilations and $(\beta\beta)_{0\nu}$ -decay in Schemes with an Inverted Mass Spectrum", S.M. Bilenky, C. Giunti, C. W. Kim and S. T. Petcov, to be published.
- [15] M. Doi, T. Kotani and E. Takasugi, Prog. Theor. Phys. Suppl. **83**, 1 (1985).
- [16] H. Ejiri, Physics and Astrophysics of Neutrinos, ed. by M. Fukugita and A. Suzuki (Springer- Verlag, Berlin, 1994).
- [17] See, for example, M. K. Moe, Nucl. Phys. B (Proc. Suppl.) **38**, 36 (1995).
- [18] B. Achkar et al., Nucl. Phys. B **434**, 503 (1995).
- [19] F. Dydak et al., Phys. Lett. B **134**, 281 (1984).
- [20] I.E. Stockdale et al., Phys. Rev. Lett. **52**, 1384 (1984).
- [21] L. Borodovsky et al., Phys. Rev. Lett. **68**, 274 (1992).
- [22] B. Armbruster et al., Nucl. Phys. B (Proc. Suppl.).
- [23] D.O. Caldwell and R.N. Mohapatra, Phys.Lett. **B354**, 371 (1995).
- [24] K.S. Hirata et al., Phys. Rev. **D65**, 2241 (1991); Y. Suzuki, Talk presented at the 6th International Workshop on Neutrino Telescopes, Venezia, March 1994.
- [25] GALLEX Collaboration, Phys. Lett.**B314**, 445 (1993); Phys. Lett.**B377** (1994).
- [26] SAGE Collaboration, Proc. of the 27th Int. Conf. on High Energy Physics, Glasgow, July 1994.
- [27] R. Davis Jr., Talk presented at the 6th International Workshop on Neutrino Telescopes, Venezia, March 1994.
- [28] J. N. Bahcall and M.H. Pinsonneault, Rev. Mod. Phys. **64**, 885 (1992). See also hep-ph/9505425.
- [29] S.P. Mikheyev and A. Yu. Smirnov, Sov. J. Nucl. Phys. **42**,913 (1985); L. Wolfenstein, Phys. Rev. **65**, 2369 (1978).